5.2: Multiplication and Division of Integers

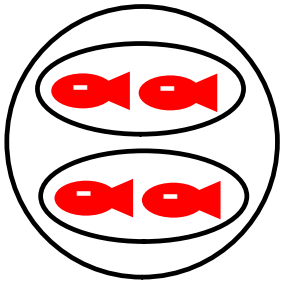
Multiplication of Integers:

Definition: For integers a and b,
(1) If $a > 0$, $a \times b = b + b + \ldots + b$ (a times).
(2) If $a = 0$, $a \times b = 0$.
(3) If $a < 0$, $a \times b = -(|a| \times b)$.

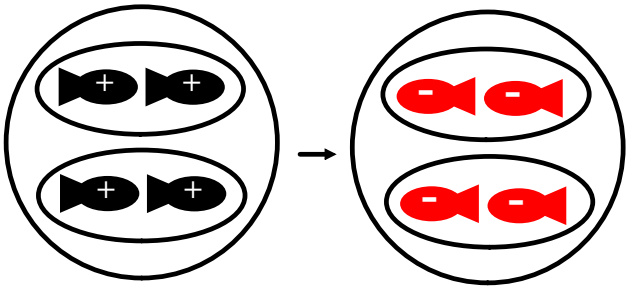
The order in this definition is more important than it may seem. The first value (ignoring negatives) is the number of groups, and the second number is what is in each group. If the first number is negative, we take the opposite after doing the multiplication.

Chip Model: Using the same setup as the chip model for addition, if $a > 0$, we represent a groups of b.

If $a < 0$, we represent $|a|$ groups of b, and then we convert to the other type of fish.



$2 \times -2 = -4$

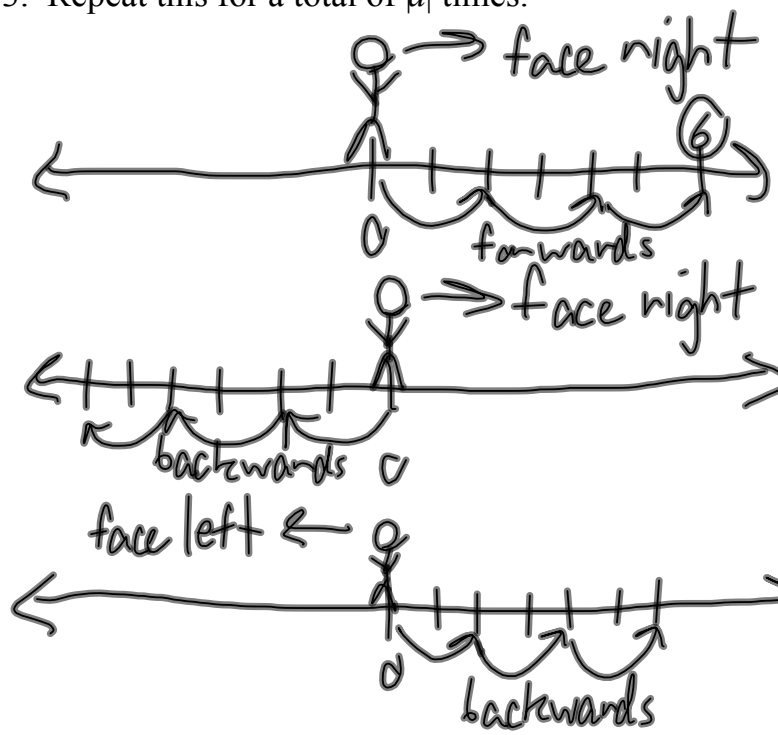


$-2 \times 2 = -4$

Charged Field Model: This model is the same as the chip model, but we use charges instead of fish.

Number Line Model: To compute $a \times b$:

1. Stand facing the direction of the sign of the first number.
2. Move the distance b forward if $b > 0$ or backward if $b < 0$.
3. Repeat this for a total of $|a|$ times.

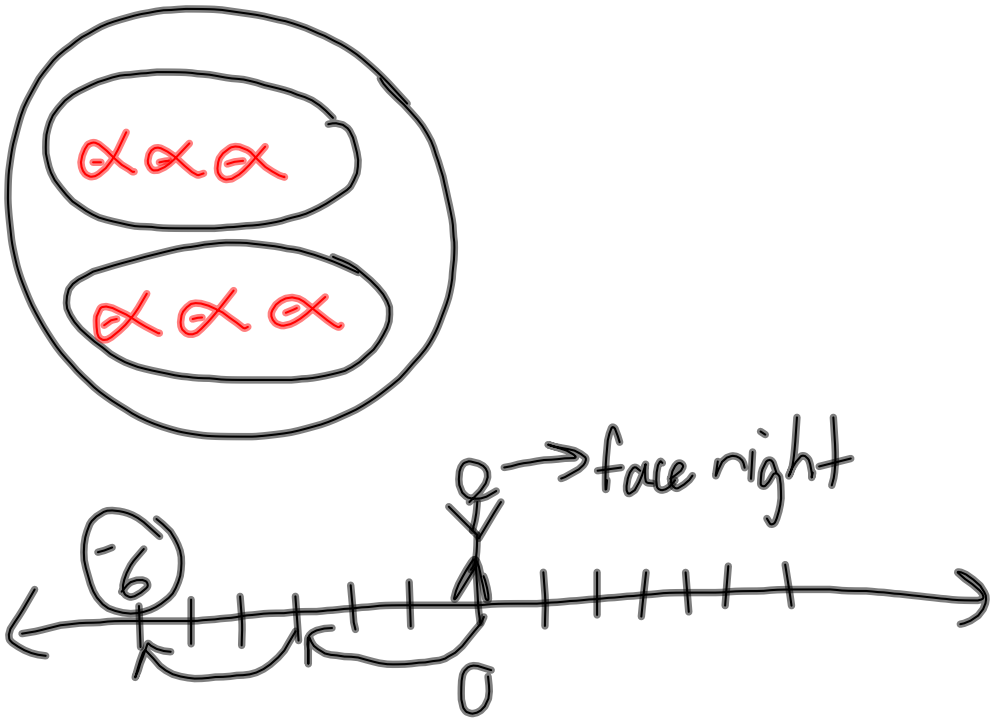


$3 \times 2 = 6$

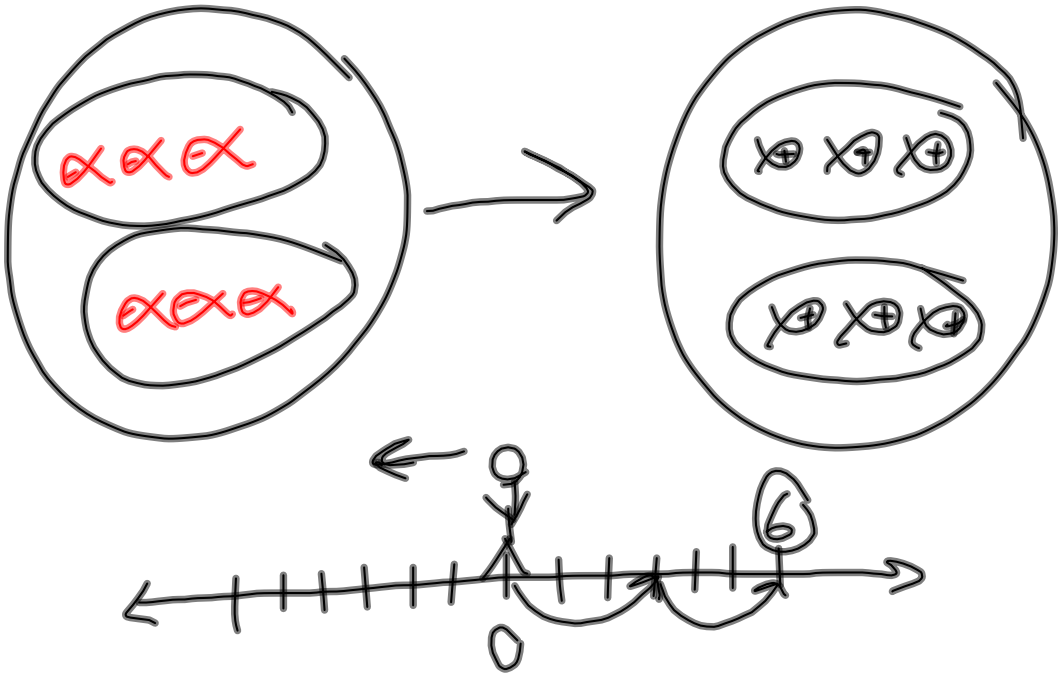
$3 \times -2 = -6$

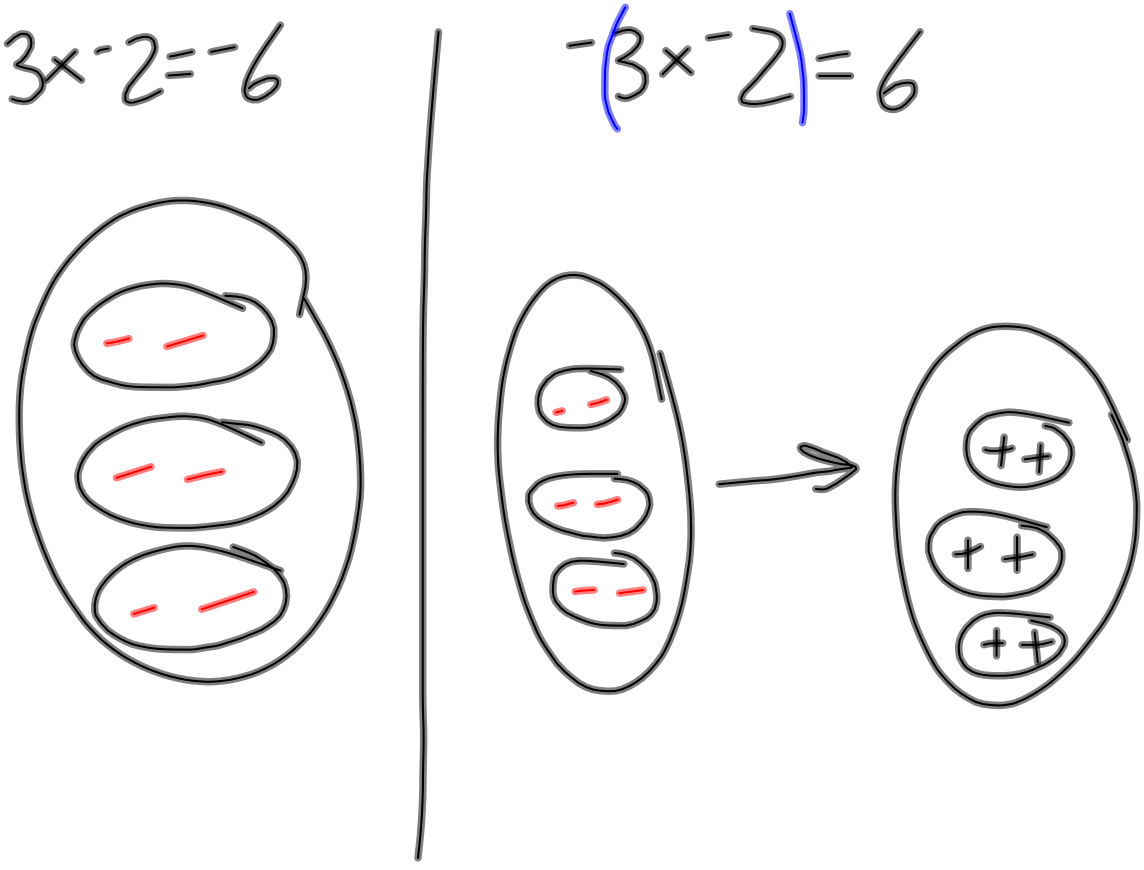
$-3 \times -2 = 6$

Example: Represent the problem $2 \times -3 = -6$ using each method.



Example: Represent the problem $-2 \times -3 = 6$ using each method.





Special Rules for Multiplication of Integers: The following set of rules can be taught to students who are struggling to speed up their calculations. This is NOT, however, a substitute for these models, as students should have a deep understanding of whether the solution is positive or negative before learning these.

1. If you are multiplying two numbers with the same sign, multiply their absolute values. (same signs positive)
2. If you are multiplying two numbers with different signs, multiply their absolute values and make this answer negative. (opposite signs negative)

Properties of Integer Multiplication:

Closure, Commutative, Associative, Identity still hold.

Distributive Property:

$a(b+c) = ab + ac$

Zero Multiplication Property:

$0 \times a = a \times 0 = 0$

Other Properties:

7. $(-1)a = -a$

5.2 Completed Notes

Division of Integers:

Definition: For integers a and b, $a \div b$ is the unique integer c, if it exists, such that $a = bc$.

Note: We say "if it exists" because $a \div b$ may not be an integer. We will not talk about a division algorithm with remainders, though it is the same with some slight restrictions on the choice of r.

Note 2: We will not learn any models for division, although a partition method with fish is a reasonable model.

Special Rules for Division of Integers: Again, this should not be taught until understanding takes place.

1. If you are dividing two numbers with the same sign, divide their absolute values. (same signs positive)
2. If you are dividing two numbers with different signs, divide their absolute values and make this answer negative. (opposite signs negative)